Is music an art or a science? Music, as an art, appeals to our emotions and intuitions. Most cultures associate music with subjective emotions that elude definition. Science is objective analysis and logic that helps us understand the material world. But does not music have the same goal as science – letting us understand the material world? While a scientist derives fundamental laws from generalizations, a musician leads us from the known to the unknown. Beethoven must have had this same thought when he defined music as the “one incorporeal entrance into the higher world of knowledge which comprehends mankind but which mankind cannot comprehend.”

A number of thinkers over the centuries have attested to the fact that music has a very close relationship to our understanding of the wider world and that the structure and grammar of music may well reflect the underlying rhythms of life. As Sir Thomas Browne (English writer and physician, 1605-82) wrote, “… there is music wherever there is a harmony, order or proportion; and thus far we may maintain the music of the spheres; for those well-ordered motions, and regular paces, though they give no sound unto the ear, yet to the understanding they strike a note most full of harmony.” And, about a hundred years before this, the Renaissance intellectual Agrippa of Nettesheim expressed similar sentiments when he wrote that “no songs, sounds and instrumental music are stronger in moving the emotions of man and in inducing magical impression than those composed in number, measure, and proportion as likenesses of the heavens.”

Let us examine the concept of emotions and proportions further. The two important pillars of every musical system in the world are: melody and harmony. Melody is variations in musical tones or pitch that produces consonance and that leads to certain emotional vibrations. Melody is usually one note following another in certain order. Harmony refers to two or more notes or tones played together; harmony is usually denoted as intervals and chords. Both harmony and melody are subject to specific arrangements of notes, chords, and other rules. That is, we can define harmony and melody through mathematics. Let us take the example of a raga, e.g. Kalyani. It has seven ascending notes and seven descending notes; and so do so many other ragas in Indian music. Why do these ragas sound different and produce different emotions? What makes them different? If instead of seven notes in the ascending order, if we remove one note, will Kalyani now sound different? What if we substitute the deleted note with another note? Would it produce another raga? Can all of these experiments be codified into an analytical table? These questions can only be answered through proper mathematical analysis, classifications, and definitions (The Melakarta scheme).

In his article on “Music, Mathematics and Bach”, Rahul Siddharthan from the Indian Institute of Science, Bangalore, highlights certain musical features (incorporating a mathematical base) which appear to be common among many cultures of the world. For example, in Western music, two notes are separated in pitch by a ‘fifth’. As Siddharthan points out, this is the interval between ‘sa’ and ‘pa’. If we choose a note a fifth above ‘pa’, we get a note of frequency 9/4 times that of the first. If we repeat this process, we could get six notes of the following frequencies: 1, 9/8, 81/64, 3/2, 27/16, 2; the series – which, mathematically speaking, is called a Pythagorean scale – represents the ‘pentatonic scale.’ In Carnatic music, it is the scale of Raga Mohanam and in Hindustani music it is Bhopali. (The fifth note in Western music is equivalent to the ‘pa’ or ‘panchama’ in the Indian tradition.)

As the mathematical illustrations point to, music is created only when there is order and proportion. Without either, there is no harmony or melody. As Sir Thomas Browne (English writer and physician, 1605-82) wrote, “… there is music wherever there is a harmony, order or proportion; and thus far we may maintain the music of the spheres; for those well-ordered motions, and regular paces, though they give no sound unto the ear, yet to the understanding they strike a note most full of harmony.” And, about a hundred years before this, the Renaissance intellectual Agrippa of Nettesheim expressed similar sentiments when he wrote that “no songs, sounds and instrumental music are stronger in moving the emotions of man and in inducing magical impression than those composed in number, measure, and proportion as likenesses of the heavens.”

May be it is time we examined how mathematics guided music or how music guided mathematics. But, we are a little late. Legend has it that the great Greek mathematician Pythagoras once ran into a forge to investigate the harmony of hammers. He noticed that most of the hammers generated a harmonious sound when struck simultaneously whereas a combination containing one particular hammer always generated an unpleasant noise. He analysed and found that those hammers which together created a harmonious sound had a simple mathematical relationship – their masses were simple ratios or fractions of each other. Hammers half, two-thirds or three-quarters the weight of a particular hammer would all generate harmonious sounds.

Pythagoras applied his new theory of musical ratios to the lyre by examining the property of a single string. He was to discover that harmonious tones only occur when the string is plucked at very specific points. For example, by fixing the string at a point exactly half-way along it, plucking generates a tone which is one octave higher and in harmony with the original tone. Similarly, fixing the string at points which were simple fractions of the whole length of the string and plucking it produced harmonious tones.

The Pythagoreans found also that the ratio 1:2 between the lengths of two vibrating strings would produce an octave, the fifth in the ratio of 2:3, the fourth in 3:4, and so on. They thus tried to understand the sublime relationship between music and numbers. Certain famous composers in the West were later to be so fascinated with this divine relationship that they would attempt to incorporate its allure in their musical structures. Claude Debussy for one was fascinated with the mathematical concept of the Golden Ratio. The Golden Ratio which is roughly equal to 1.618033989 is the decimal value that the fractions of adjacent Fibonacci numbers head towards. (The Fibonacci sequence is an unending series of numbers in which each number, except for the first two, is the sum of the preceding two.) In Debussy’s “Prelude to the Afternoon of a Faun”, the Golden Ratio can be perceived in pitch, rhythm and dynamics. Its musical pulses (also called quaver units) correspond to a pattern. For instance, they incorporate fortissimos (or ff, very loud) at bars 19 and 28 and then the next fortissimo at bar 47 which is the sum of 19 and 28. The piece contains also a number of similar patterns which follow the Fibonacci rules.

More than 2000 years later, Johannes Kepler concluded however that the Pythagoreans actually were wrong in trying to explain musical harmony using numbers. Kepler stated that the explanation should instead be sought using geometry. He imagined a string to be bent into the form of a circle which can be divided in equal parts by drawing symmetrical figures with varying numbers of sides inside it. The drawing of a pentagon will thus divide the circumference into parts of 1/5 and 4/5 – and these will correspond to consonant chords. But a heptagon’s sides will produce ratios of 1/7 and 6/7 which equate to discords. This, according to Kepler, was because the pentagon can be constructed using a compass and ruler, but not the heptagon. In Kepler’s opinion, geometry is the only language which enables man to understand the working of the divine mind. Therefore figures that cannot be constructed by compass and ruler – the only permissible tools in classical geometry – were somehow “unclean, unknowable, unspeakable, non-existences.”

To summarize, it is irrelevant whether music guided mathematics or mathematics guided music. When we view music at a more spiritual level, this instinctual awareness of how certain mathematical ratios appear to foster brilliant music perhaps reiterates the contention that tonal quality and rhythms are nothing more than a reflection of the fundamental heartbeat of nature that enhances our perception of the world around us. As Gottfried Leibnitz observed, music is “a kind of counting performed by the mind without knowing that it is counting”. It hence does not matter how we become aware of nature’s heartbeat – it can be either directly via the numbers themselves or via the more graphic medium of their geometry.